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TRANSMISSION LINE VIBRATION WITH DAMPERS

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1. Introduction

The relation between the frequency of conductor vibration and the velocity of the wind blow observed in the field span gives a resulting graph (figure 1) in which the build up ratio of the conductor frequency is higher than that of wind velocity especially in the domain of high wind velocity. This phenomena could be explained by taking the flexural rigidity of the conductor into account.

The equation to write the wave form of the vibrating conductor, which is controlled by its own flexural rigidity, will be obtained from the superposition of the sinusoidal and hyperbolic functions.

Furthermore, the equation to write the vibration amplitudes will be introduced by using the boundary conditions peculiar to each of the subspans terminated with the span extremities and damper locations.

It is well known that when there is no damper the value of the dynamic strain caused by the conductor vibration is equivalent to the product of vibration frequency (f_0) and double amplitude of the vibration (Y_0), that is equal to $f_0 \cdot Y_0$.

And, when there are some dampers, it is required that the product of frequency (f) and amplitude (Y) is to be multiplied once again by a coefficient " $\varepsilon_i(\rho)/f$" which is called "Strain Coefficient" in this paper.

The aim of this investigation is to study the effect of the strain coefficient ($\varepsilon_i(\rho)/f$) on the dynamic strain (ε) caused by the vibration of the conductor (IACSR120mm²) with three ideal dampers clamped near the span extremities.

2. Wave form of vibrating conductor with no damper

The equation of conductor vibration is described as :

$$M\frac{\partial^2 y}{\partial t^2} = S\frac{\partial^2 y}{\partial x^2} - EI\frac{\partial^4 y}{\partial x^4} \qquad (1)$$

S = tension in conductor, N.

M = mass per unit length of conductor, Kg/m.

EI = flexural rigidity of the conductor, N-m².

where flexural rigidity of the conductor was introduced as an important element [1].

And, this is solved to get the equation suitable for writing the wave form along the conductor, that is

$$Y_H = M_1 \cos \mu_2 x + M_2 \sin \mu_2 x + M_3 \cosh \mu_1 x + M_4 \sinh \mu_1 x \qquad (2)$$

$$\mu_2 = 2\pi/\lambda \qquad \lambda = \text{wave length, m.}$$

$$\mu_1 = \sqrt{\mu_2^2 + S/EI} \qquad 1/\text{m} \qquad (3)$$

where, put $f = \frac{\mu_2 \cdot \mu_1}{2\pi} \cdot \sqrt{EI/M}$, Hz

C_0 = wave velocity along the conductor when $EI = 0$

C = wave velocity along the conductor when $EI \neq 0$

Then, by the use of equation (3), there comes

$$C_0 = \lambda f_0 = \sqrt{S/M}$$

$$C = \lambda f = \mu_1 \cdot \sqrt{EI/M} \qquad (4)$$

$$f/f_0 = C/C_0 = \mu_1 \cdot \sqrt{EI/S} = \sqrt{\mu_2^2 + S/EI} \cdot \sqrt{EI/S} = \sqrt{1 + \mu_2^2 \cdot EI/S} \qquad (5)$$

The above equation may be rewritten as:

$$f/f_0 = C/C_0 = \sqrt{1 + \omega_0^2 \cdot M \cdot EI/S^2} \fallingdotseq 1 + \omega_0^2 \cdot M \cdot EI/2S^2 \qquad (5)'$$

where,

$$\mu_2 = 2\pi/\lambda = \omega_0/C_0 = \omega_0 \cdot \sqrt{M/S}$$

$$\mu_2^2 \cdot EI/S = \omega_0^2 \cdot M \cdot EI/S^2$$

The same equation can be seen in the CIGRE's paper [2].

For example :

with respect to the conductor (IACSR120mm^2)

EI = 344.6 N-m^2

S = 21854 N

and $f/f_0 = 0.1256\mu_1$ (see table 1) (6)

There is one of the experimental results showing the relation between wind velocity and vibration frequency. This is plotted in the figure 1, where the wellknown equation ($f_0 = 0.2 \cdot V/D$) is true for low wind velocity, but it is not true for high wind velocity.

For high wind velocity, equation (6) is of use to write the vibration frequencies as are shown in the figure. Where, it would be recognized that the flexural rigidity of the vibrating conductor plays an important role for the vibration problems.

Now, by the use of flexural rigidity (EI), the equation to write the wave form of vibration amplitude (Y_M) will be expressed as well as equation (2).

3. Wave form of vibrating conductor with dampers

There is left half of the one span as is shown in figure 2 ; where,

N = center of the whole span $\overline{CC'}$

C = span termination of left-hand side.

D = damper location.

Y_1 = vibration amplitude along the subspan $\overline{CD}$, where $\overline{CD} = a$

Y_2 = vibration amplitude along the subspan $\overline{DN}$, where $\overline{DN} = b$, $a + b = l$

f_D = force applied to the conductor at the damper location.

The boundary conditions around the half span, when the antinode of the wave form comes upon the center N, are described as follows :

Point C (span termination)

$$(Y_1)_{x=0} = 0$$

$$\left(\frac{dY_1}{dx}\right)_{x=0} = 0 \qquad (7)$$

Point N (span center)

$$\left(\frac{dY_2}{dx}\right)_{x=b} = 0$$

$$\left(\frac{d^3Y_2}{dx^3}\right)_{x=b} = 0 \qquad (8)$$

Point D (damper location)

$$(Y_1)_{x=a} = (Y_2)_{x=0}$$

$$\left(\frac{dY_1}{dx}\right)_{x=a} = \left(\frac{dY_2}{dx}\right)_{x=0}$$

$$\left(\frac{d^2Y_1}{dx^2}\right)_{x=a} = \left(\frac{d^2Y_2}{dx^2}\right)_{x=0} \qquad (9)$$

$$EI\cdot\left[\left(\frac{d^3Y_2}{dx^3}\right)_{x=0} - \left(\frac{d^3Y_1}{dx^3}\right)_{x=a}\right] = f_D$$

For the present study, dampers clamped near the span extremities are ideal ones of which force applied to the conductor (f_D) will be given as :

$$f_D = \alpha \cdot k \cdot \omega \cdot Y$$

$$\alpha = 1.0$$

$$k = \sqrt{S \cdot M} \qquad (10)$$

$$\omega = \mu_1 \cdot \mu_2 \cdot \sqrt{EI/M}$$

$$\xi = \mu_1/\mu_2$$

If there are three dampers for the left half span as is shown in the figure 3, then equation (9) will be applied to each of the three damper locations (D, E, F). And, from these boundary conditions inclusive of equation (7) and equation (8) the equation to write the vibration amplitudes (Y_M ; M = C, D, E, F) will be introduced. For numerical calculations refer to the "Calculation Formulas" in the appendix 1.

4. Strains on the Conductor

a) With no dampers

The standing wave along the conductor (Y_M) is

$$Y_M = \cos\mu_2 x - \frac{\mu_1}{\mu_2}\sin\mu_2 x - \cosh\mu_1 x + \sinh\mu_1 x \qquad (11)$$

provided that $M_1 = 1.0$

And the strain $(\varepsilon)_1$, per 1 mm of the sinusoidal amplitude $|SY_M|$ is equal to

$$(\varepsilon)_1 = K_2 \cdot \frac{1}{\rho} \cdot \frac{D}{2} \Big/ |SY_M|$$

$$= -K_2 \cdot \frac{D}{2} \cdot (\mu_2^2 + \mu_1^2) \Big/ \sqrt{1 + (\mu_1/\mu_2)^2}$$

$$= -K_2 \cdot \frac{D}{2} \cdot \mu_2 \cdot \sqrt{\mu_2^2 + \mu_1^2} \qquad (12)$$

For the low frequencies, put as follows :

$$\lambda f_0 = \sqrt{S/M}$$

$$\mu_2 = 2\pi/\lambda$$

$$\sqrt{\mu_2^2 + \mu_1^2} \fallingdotseq \mu_1 \fallingdotseq \sqrt{S/EI}$$

then

$$(\varepsilon)_1 = K_2 \cdot \frac{D}{2} \cdot \frac{2\pi}{\lambda} \cdot \sqrt{S/EI}$$

$$= (K_2 \cdot \pi \cdot D \cdot \sqrt{M/EI}) \cdot f_0 \qquad (13)$$

and, for the double amplitude (Y_0), p-p, mm, the value of strain ($\pm\varepsilon_i$), when $K_2 = 0.7$, will be written as

$$|\varepsilon_i| \fallingdotseq \left(\frac{K_2}{2} \cdot \pi \cdot D \cdot \sqrt{M/EI}\right) \cdot (f_0 \cdot Y_0) \qquad (14)$$

or $|\varepsilon_i| \fallingdotseq f_0 \cdot Y_0$ ε_i, ± h-p ; Y_0, p-p (15)
because of

$$\frac{K_2}{2}\cdot\pi\cdot D\cdot\sqrt{M/EI} = \frac{0.7\times3.14\times17.5}{2\times20.1872} = 0.953 \fallingdotseq 1.0$$

From another point of view, it is worth while to introduce the "Strain Coefficient" ($\varepsilon_i(\rho)/f$) which is equal to the strain per unit double amplitude ($(\varepsilon)_i/2$) for unit frequency in Hz. That is,

$$\varepsilon_i(\rho)/f = |(\varepsilon)_i|/2f$$
$$= K_2\cdot\frac{D}{2}\cdot\mu_2\cdot\sqrt{\mu_2^2+\mu_1^2}\Big/\left[2\cdot\frac{\mu_1\mu_2}{2\pi}\cdot\sqrt{\frac{EI}{M}}\right]$$

and, for low frequency,

$$\varepsilon_i(\rho)/f = \frac{K_2}{2}\cdot\pi\cdot D\cdot\sqrt{\frac{M}{EI}}\cdot\frac{\sqrt{\mu_2^2+\mu_1^2}}{\mu_1}$$
$$\fallingdotseq \frac{K_2}{2}\cdot\pi\cdot D\cdot\sqrt{M/EI} \fallingdotseq 1.0 \qquad (16)$$

So that, for low frequencies, equation (15) may be written as follows :

$$|\varepsilon_i| = (\varepsilon_i(\rho)/f)\cdot(f_0\cdot Y_0)$$
$$\varepsilon_i(\rho)/f = \left[K_2\cdot\frac{1}{\rho}\cdot\frac{D}{2}\Big/|SY_M|\right]\Big/f_0 \qquad (17)$$
$$\varepsilon_i(\rho)/f = 1.0$$

b) With dampers

In figure 3 which shows the left half of the vibration conductor with three dampers, there are four subspans where the amplitudes are equal to Y_M ($M = C, D, E, F$).

The strain and strain coefficient should be discussed with respect to the main subspan Y_F. In accordance with above thoughts, it seems very natural to put as follows :

$$|\varepsilon_i| = (\varepsilon_i(\rho)/f)\cdot(f\cdot Y)$$
$$\varepsilon_i(\rho)/f = \left[K_2\cdot\frac{1}{\rho}\cdot\frac{D}{2}\Big/|SY_F|\right]\Big/f \qquad (18)$$
$$\varepsilon_i(\rho)/f \neq 1.0$$

Where it will be accepted to take the value of $1/\rho$ derived from Y_C, because the values of $1/\rho$ corresponded to the points of damper locations may be very small for the use of β_1 and β_2 (see appendix 1). Because, these values of β_1 and β_2 are selected so as to make the hyperbolic components of Y_M ($M = D, E$) as small as possible.

The following is obtained from the equation (14) by using the relation written in equation (4).

$$|\varepsilon_i| = (\frac{K_2}{2}\cdot\pi\cdot D\cdot\sqrt{M/EI}\cdot f_0)\cdot Y_0$$
$$= (\frac{K_2}{2}\cdot\pi\cdot D\cdot\sqrt{M/EI}\cdot\frac{1}{\lambda}\cdot\sqrt{S/M})\cdot Y_0$$
$$= K_2\cdot\pi\cdot\sqrt{S/\mathrm{EI}}\cdot\frac{Y_0}{2}\cdot\frac{1}{\lambda}\cdot D$$
$$\fallingdotseq K_2\cdot3.27\cdot\sqrt{S/\mathrm{EI}}\cdot\frac{Y_0}{2}\cdot\frac{1}{\lambda}\cdot D \qquad (19)$$

One can find the same equation in the CIGRE's paper [2]. The use of numerical value equal to 3.27 instead of π may give the best approach compared with the actual phenomenon.

Numerical caluclations in this paper will be proceeded after the same idea, taking as follows :

$$K_2/2 = 0.7/2 = 0.35 \longrightarrow \frac{3.27}{\pi}\times\frac{K_2}{2} = 0.36$$

or $\varepsilon_i(\rho)/f = \left[0.36\cdot17.5\cdot\frac{1}{\rho}\Big/|SY_F|\right]\Big/f$ (18)'

Moreover, CIGRE's paper [2] says ;

1) Equations are valid for rigidly clamped span extremities which means that neither vertical displacement nor any rotation of the cable extremities is allowed.
2) Real conditions might differ from the "rigid clamp" assumption, resulting in lower strains and sometimes also in higher strain values.

And, as mentioned, the contents of equation (11) is correspondent to equation (17) and the contents of equation (2) is correspondent to equation (18).

5. Numerical calculations for test span

The numerical study will be used only upon the conductor IACSR120mm² with three ideal dampers fixed at each end of the span.

a) Resonant frequency

From equations (3) and (4),

$$f_0 = 0.2V/D = 11.4286V \qquad D = 0.0175$$
$$\mu_2 = 2\pi/\lambda = 2\pi f_0/C_0$$
$$= (2\pi\cdot0.2V/D)\cdot\sqrt{M/S} = 0.4467V$$
$$\mu_1 = \sqrt{\mu_2^2 + 63.4185}$$
$$f = 3.2129\mu_1\cdot\mu_2$$

Thus, the values shown in table 1 are determined.

Table 1

V	f_0	μ_2	μ_1	f	f/f_0
0.5	5.71	0.223	7.967	5.71	1.000
1.0	11.43	0.447	7.976	11.45	1.002
2.0	22.86	0.893	8.014	22.99	1.006
5.0	57.14	2.234	8.271	59.37	1.039
10.0	114.29	4.467	9.131	131.05	1.147
12.5	142.86	5.584	9.726	174.49	1.222
15.0	171.43	6.701	10.408	224.08	1.307

Following the increase of wind velosity (V) the value of frequency (f) increases considerably higher than that of f_0 and the ratio f/f_0 which is 1.00 when $V = 0.5$ m/sec attains 1.31 when $V = 15$ m/sec.

The relationship between wind velosity (V) and vibration frequency (f) is apparent in figure 1 where the tested results at Tsuruga Line in Japan (1977) are plotted, and moreover the characteristic curves for f_0 and f are added to them. It is conceived that the locus of f shows a closer connection to the experimental results plotted in figure 1.

The above terms imply that the effect of "Flexural Rigidity" (EI) cannot be neglected to handle the problems connected to the conductor vibrations.

Here, it must be remembered that in the CIGRE's paper [2], there are written as follows :
1) the range of wind velocity is about 0.5~10 m/s
2) the range of vibration frequency is about 3~120 Hz

But, in figure 1, these upper limits are extended up to 1.5 times of themselves.
That is,

$$V = 10 \text{ m/sec} \longrightarrow 15 \text{ m/sec}$$
$$f = 120 \text{ Hz} \longrightarrow 180 \text{ Hz}$$

And, for the original value of dynamic strain $\|\varepsilon_i\|$ correspondent to the approximate value of $|\varepsilon_i|$ in equation (19), it is necessary to apply the following calculation :

$$\|\varepsilon_i\| = K_3 \cdot |\varepsilon_i|$$
$$K_3 = \sqrt{\mu_2^2 + \mu_1^2} / \sqrt{S/EI} = \sqrt{\mu_2^2 + \mu_1^2} / 7.9636$$

There are calculated values in table 2. The value of K_3 grows greater and greater with the increase of frequency domain,

Table 2

f	μ_2	μ_1	$\sqrt{\mu_2^2 + \mu_1^2}$	K_3
45.01	1.7195	8.1471	8.3266	1.0456
60.02	2.2570	8.2772	8.5794	1.0773
75.00	2.7688	8.4312	8.8742	1.1143
90.07	3.2580	8.6042	9.2004	1.1553
105.00	3.7185	8.7890	9.5433	1.1984
120.07	4.1595	8.9844	9.9005	1.2432
135.02	4.5755	9.1844	10.2610	1.2885
150.05	4.9740	9.3893	10.6254	1.3342
165.00	5.3523	9.5919	10.9869	1.3796
180.00	5.7154	9.8023	11.3468	1.4248

b) Vibration amplitude

The equation to write the wave form of vibration amplitude (Y_M) is expressed as a function of various parameters such as span length, position of damper location, and force transmitted to the conductor for the presence of the dampers. Calculations will be done after the "Calculation Formulas" described in the appendix 1.

To get the values of wave coefficient Mn (n = 1. 2. 3. 4) for each of the vibration amplitude Y_M ($M = C, D, E, F$) the span length (L_R) was selected as :

$$L_R = 2l = 100\text{m} \qquad (\text{see figure 3}) \quad (20)$$

And, to get the amplitude for balanced condition of energies, the span length (L_F) was selected as :

$$L_F = L_R + n\lambda \fallingdotseq 800\text{m} \qquad n : \text{integer} \quad (21)$$

where, it is expected that the same boundary conditions may be applied for both equations (20) and (21).

c) Dynamic strain

The strain caused by the vibration of the conductor with dampers will be written as :

$$|\varepsilon_i| = (\varepsilon_i(\rho)/f) \cdot (f \cdot Y)$$
$$\varepsilon_i(\rho)/f \neq 1.0 \qquad (\text{with damper}) \qquad (18)'$$

or

$$|\varepsilon_i| = f \cdot [(\varepsilon_i(\rho)/f) \cdot Y]$$
$$= f \cdot \alpha Y \qquad (22)$$
$$\alpha Y = (\varepsilon_i(\rho)/f) \cdot Y$$

If there is no damper, then the equation (17) turns into

$$|\varepsilon_i| = f_0 \cdot Y_0 \qquad (\text{without damper}) \qquad (17)'$$

That is, the value of αY in equation (22) is correspondent to the value of Y_0 in equation (17)'. So that, αY may be called "Equivalent Amplitude".

For the second step, numerical calculation will be done from one frequency to the another. The calculated data are marked with :

× ; at intervals of 5 Hz
○ ; at intervals of 5/4 = 1.25 Hz
△ ; at intervals of 5/8 = 0.625 Hz

on the datum papers.

In figure 4, there is calculated value of the strain (ε) as a function of energy balance amplitude ($2Y_B$). The plotted points are scattered from place to place giving various values of the strain for one value of the amplitude.

Moreover, the calculated relations between f and ε, and between f and αY are as shown in figure 5 and figure 6. The numbers noted as I, II, III, IV, in each of the figures mean that the starting frequency of the loci has a lag of 1.25 Hz for every step of I → II, II → III, III → IV.

There are close resemblance between the couples (I, I), (II, II), etc., because of $\varepsilon = f \cdot \alpha Y$. But, there is no connection between different numbers. And, this result means that it is not advisable to draw the envelope of dynamic strains based on the data calculated at intervals of 5 Hz.

Such phenomenon as mentioned above is originated that the value of strain coefficient $(\varepsilon_i(\rho)/f)$ has no simple connection with the frequency (f).

Figure 7 shows the relation $f : \varepsilon_i(\rho)/f$ and is plotted by the use of the following marks :

× ; at intervals of 5 Hz for 5~150 Hz
○ ; at intervals of 1.25 Hz for 20~90 Hz
△ ; at intervals of 0.625 Hz for 80~85 Hz

where, the points of ×-mark are connected from a point to the next point with straight lines for the better sight to catch the ×-points.

The reader who has seen the above figure at a glance may catch the scattered points (○-mark and △-mark) plotted for the values of strain coefficient $(\varepsilon_i(\rho)/f)$.

The allowable value of vibration amplitude for the safety strain (±150 MICRO) will be expressed as follows :

1) allowable amplitude $(\overline{\alpha Y})$ without damper

$$\overline{\alpha Y} = 150 f^{-1} \qquad (24)$$

2) allowable amplitude $(\overline{2Y_B})$ with dampers

$$\overline{2Y_B} = \overline{\alpha Y}/(\varepsilon_i(\rho)/f) \qquad (25)$$

The calculated results by the use of equation (25) are plotted in the figure 8. And, it will be found that,

for the region of 45~75 Hz

$\varepsilon_i(\rho)/f < 1.0$ in figure 7
$\overline{2Y_B} > \overline{\alpha Y}$ in figure 8

thus, it is determined that

$$\text{if } \varepsilon_i(\rho)/f < 1.0 \text{ then } \overline{2Y_B} > \overline{\alpha Y} \qquad (26)$$

The equation (26) tells that for the above mentioned frequency domain the allowable amplitude without damper $(\overline{\alpha Y})$ will be increased up to the value of $\overline{2Y_B}$ when dampers are installed. Where, high amplitude makes low strain.

Now, there is also another frequency domain :

$$\text{if } \varepsilon_i(\rho)/f > 1.0 \text{ then } \overline{2Y_B} < \overline{\alpha Y} \qquad (27)$$

Equation (27) tells that when $\varepsilon_i(\rho)/f > 1.0$ it is recommended that by the use of dampers the vibration amplitude must be decreased below $\overline{2Y_B}$ which is less than $\overline{\alpha Y}$. Where the low amplitude makes high strain if the vibration amplitude retains between $\overline{2Y_B}$ and $\overline{\alpha Y}$.

6. Strains on a field span

On the basis of energy balance principle calculations will be conducted. For these matters the span length was selected as long as a field span, that is equal to about 800m (see equation (21)). Three dampers are placed near each of span extremities.

For the simple calculation, the wind powers applied to the subspans $(\overline{CD}, \overline{DE}, \overline{EF})$ and the damping powers consumed by the subspans $(\overline{CD}, \overline{DE}, \overline{EF})$ per unit length of the conductor (see figure 3) were assumed to be equal to the power applied and/or consumed by the main subspan $(\overline{FN})$.

Put,

$C(f)$ = damping power consumed by the conductor per unit vibration amplitude (peak to peak)

$W(f)$ = wind power applied to the conductor per unit vibration amplitude (peak to peak)

$(2W)_1$ = total power consumed by the dampers per unit vibration amplitude (peak to peak)

Then, the energy balance amplitude $(2Y_B)$ will be gotten by using the following equations.

$$C(f) \cdot (2Y_B)^{1.305} + (2W)_1 \cdot (2Y_B)^{0.805} = W(f)$$
$$C(f) = 2.5218 \times 10^{-12} \cdot f^5 \cdot 800 \qquad (28)$$
$$W(f) = 7.8504 \cdot C_L^2 \times 10^{-8} \cdot f^3 \cdot 800 \cdot 0.7$$
$$C_L^2 = 0.1 + 1.6/[1 + 0.08(1 - 5 \cdot 0.0175 \cdot f)^2]$$

Equation (28) will be used in this paper for higher frequency domain although there is no experimental data to verify that the above equations are acceptable.

Calculated value of balance amplitudes is plotted as a function of frequency in figure 9. The figure displays that the plotted points draw some irregular and frequent repetition going up and down. Almost all the maximum values correspondent at the intervals of 1.25 Hz (○-mark) are involved within a descending line which is nearly straight. The maximum value of $2Y_B$ is equall to 13.7 mm correspondent to the frequency of 28.75 Hz. On the other hand, below the above descending line there are points of far low values which scatter over the whole frequency range.

The value of strains when the vibration amplitude attained to the energy balance condition are shown in figure 10. There are two peak values of dynamic strain (ε).

On the first peak ;

dynamic strain, $\varepsilon \fallingdotseq 90$ MICRO
frequency band, $f = 20 \sim 45$ Hz
where $\varepsilon > 50$ MICRO

On the second peak ;
dynamic strain, $\varepsilon \fallingdotseq 230$ MICRO
frequency band, $f = 75 \sim 90$ Hz
where $\varepsilon > 80$ MICRO

That is, the figure drawn concerning the strain (figure 10) is far from that drawn concerning with the balance amplitude (figure 9).

On the contrary, figure 11 drawn with accord to the "Equivalent Amplitude" (αY) resembles closely the figure 10 drawn with accord to the dynamic strain (ε).

The relation $\alpha Y > \overline{\alpha Y}$ shown for the same frequency domain in figure 11 is correspondent to the relation $\varepsilon > \bar{\varepsilon}$ shown in figure 10. Where the value of $\bar{\varepsilon}$ means the value of allowable limit that is equal to ± 150 MICRO.

The loci of which calculated points are connected step by step by the use of straight lines to display the values of balance amplitude ($2Y_B$) and strain coefficient ($\varepsilon_i(\rho)/f$) as function of vibration frequency (f) are shown in figure 12. The domain of frequency selected was $80 \sim 85$ Hz. Also, there is a locus of dynamic strains (ε) which are given as ;

$$\varepsilon = (\varepsilon_i(\rho)/f) \cdot (2Y_B) \cdot f$$

When the locus of $2Y_B$ goes upward, the locus of $\varepsilon_i(\rho)/f$ goes downward, and vice versa. This is why the locus of $2Y_B$ is not directly connected to the locus of ε as is apparently shown in figure 12.

Draw a smooth envelope which goes along the calculated peak value of strains, then there comes out such points which are situated far below the above envelope as are expressed as s.p in the figure 12. There are the same points scattered in figure 10.

The points (○-mark and ×-mark) situated nearly on the envelope of dynamic strain (ε) in figure 10 are enclosed with squares for the better sight and expressed by $(\varepsilon)_\square$. Let the frequency correspondent to $(\varepsilon)_\square$ in the figure 10 be $(f)_\square$. Again, concerned above, the balance amplitude ($2Y_B$) and strain coefficient ($\varepsilon_i(\rho)/f$) are also expressed by $(2Y_B)_\square$ and $(\varepsilon_i(\rho)/f)_\square$ as can be seen in figure 9 and figure 7.

As long as $(\varepsilon)_\square$ concerned there are following relations :

1) $(2Y_B)_\square$;

a. When $f < 55$ Hz, $(2Y_B)_\square$ are always equal to the maximum value compared to the value $2Y_B$ for $(f)_\square \pm \triangle f$
$\triangle f = 1.25$ Hz
and the effects of EI are little because $f/f_s < 1.03$

b. When $f > 55$ Hz, $(2Y_B)_\square$ are not always equal to the maximum balance amplitude for $(f)_\square \pm \triangle f$
and the effects of EI are great.

2) $(\varepsilon_i(\rho)/f)_\square$;

a. When $f < 75$ Hz ; $(\varepsilon_i(\rho)/f)_\square < 1.0$ and $(\overline{2Y_B})_\square > (\overline{\alpha Y})_\square$ (see equation 26)

b. When $f > 75$ Hz $(\varepsilon_i(\rho)/f)_\square > 1.0$ and $(\overline{2Y_B})_\square < (\overline{\alpha Y})_\square$ (see equation 27)

All of the preceding matters are based upon the calculation accomplished by the use of equations (see appendix 1) obtained for a conductor (IACSR120mm^2) with three ideal dampers.

7. Dampers

a) Conventional damper ;

As stated, there is no welcome to use the ideal dampers because they have strains (ε) such as

$\varepsilon > 150$ MICRO

between the frequency band from 80 Hz up to 85 Hz.

The force transmitted from a conventional damper to the conductor (f'_D) will be written compared to the force transmitted to the conductor by an ideal damper (f_D).

That is,

$$f'_D = \alpha(f) \cdot f_D$$
$$\alpha(f) \neq 1.0 \qquad (29)$$

The value of $\alpha(f)$ varies according to the frequency. For the calculations, the value K' (see appendix 1 ; B)—11) is variable and consequently the value related to the wave form of the conductor vibration (Y_M: $M = C, D, E, F$) is variable because these are functions of $\alpha(f)$.

And, K' may be written in the form :

$$K' = \alpha \cdot k \cdot \omega / [EI \cdot \mu_2^3 \cdot (1 + \xi^2)] \quad \alpha = 1.0$$
$$= 7.9636\xi / [\mu_2(1 + \xi^2)]$$

Here, $\alpha \cdot k \cdot \omega$ is proportional to the force applied to the conductor. And, when $\alpha \neq 1.0$, K' is to be multiplied by α (see equation (10)).

The previous statement will give the results as

Table 3 ($f = 81.25$ Hz)

$\alpha(f)$	K'	$\varepsilon_i(\rho)/f$	$2Y_B$	$\overline{2Y_B}$	αY	$\overline{\alpha Y}$	ε
1.0	0.8346	8.053	0.346	0.229	2.785	1.846	226.3
1.2	1.0015	7.518	0.337	0.246	2.532	1.846	205.7
1.5	1.2519	6.879	0.325	0.268	2.234	1.846	181.5
2.0	1.6692	6.115	0.309	0.302	1.888	1.846	153.4

are shown in table 3 for the conductor IACSR120mm² with 3-dampers, while the vibration frequency keeps 81.25 Hz.

The results will be noted that accompanied with the decrease of $\varepsilon_i(\rho)/f$

$2Y_B$ comes near by $\overline{2Y_B}$, and

αY comes near by $\overline{\alpha Y}$

so as ε has decreased to 226 → 153.

If the function of $\alpha(f)$ is well known for wide domain of frequency, the same calculations as are in the table 3 can be completed for the practical use.

When the results of above calculation are not successful, it is advisable to do the calculation again by the use of another distance for the installation of dampers (there is no necessity to put $c = d = e$ in figure 3).

b) Powers consumed ;

Dampers are installed to the conductor at the points D, E, F, near each of the conductor extremities as are shown in figure 3. The powers consumed in each couple of the dampers for both extremities are expressed as

$\Delta W\text{-}D$, $\Delta W\text{-}E$, $\Delta W\text{-}F$

And, the results of calculated powers are given in table 4 (see appendix 2) in which the calculation was made at the frequency intervals of 1.25 Hz for the frequency domain 20～90 Hz.

Moreover, on the numerical value correspondent to the maximum for each of the frequency, there is an under-line. As a total summation there are :

16 lines which are correspondent to point D,
6 lines 〃 point E,
35 lines 〃 point F.

Refering to the above, one can set the rank of probability of fatigue as :

1 st, dampers installed at the point F,
2nd, 〃 point D,
3rd, 〃 point E.

The same work can be accomplished for any damper when there are values of $\alpha(f)$ in equation (29) for given domain of frequency.

c) Another equation for $\varepsilon_i(\rho)/f$:

When there is no damper, the sinusoidal component of the vibration amplitude is equal to SY_C. But, when dampers are installed the value of SY_C goes up or down to catch the value SY_F which is the sinusoidal component of the main subspan ($\overline{FN}$) in figure 3. This is why the value of "Strain Coefficient" takes various values dependent on the frequency. And, the above mentioned things will be recognized when the relationship between equation (17) and equation (18) will be clarified.

The value of strain will be multiplied about SY_C/SY_F times according to the change of $SY_C \rightarrow SY_F$. The results of calculations are

$$\varepsilon_i(\rho)/f = \gamma(f)\cdot(\sqrt{1+\xi^2}/SY_F)$$
$$\sqrt{1+\xi^2} = SY_C$$
$$\gamma(f) = 0.98 \sim 1.11$$

as is shown in table 5.

Table 5

f	$\varepsilon_i(\rho)/f$	$\sqrt{1+\xi^2}/SY_F$	$\gamma(f)$
5.00	2.1894	2.2308	0.981
25.00	0.3212	0.3252	0.988
50.02	0.4583	0.4554	1.006
75.00	1.3676	1.3252	1.032
100.02	3.3565	3.1688	1.059
125.05	1.3137	1.2102	1.086
150.05	0.8566	0.7720	1.110

8. Conclusions

During this study there were introduced the following matters for the handling of vibration problems :

1) Equations Y_M ($M = C, D, E, F$)
2) Flexural Rigidity (EI)
3) Strain Coefficient ($\varepsilon_i(\rho)/f$)

Each of the Y_M consists of four wave coefficients (Mn; $n = 1, 2, 3, 4$) which are based upon the boundary conditions. The effect of EI is predominant peculiare to the higher domain of vibration frequency. And, the value of $\varepsilon_i(\rho)/f$ is directly connected to the value of dynamic strain on the conductor with dampers.

The results introduced after calculations are :

1) The relation between wind velocity and vibration frequency is written as

$f_0 = 0.2V/D$

This is almost sharp true for low frequencies, but the frequencies build up more rapidly than the wind velocities for higher frequencies. And the latter frequencies are written as

$f = (\mu_0 \cdot \sqrt{EI/S}) \cdot f_0$

which is dependent on the value of flexural rigidity of the conductor (EI).

2) There are four equations to express the vibration amplitudes, each of which is made as the result of installation of three dampers.

When there is no damper, the dynamic strain (ε) at the extremity of the conductor is equal to

$\varepsilon = (\varepsilon_i(\rho)/f) \cdot (f_0 Y_0)$

$\varepsilon_i(\rho)/f = 1.0$

But, when there are dampers, the above equation goes to

$\varepsilon = (\varepsilon_i(\rho)/f) \cdot (fY)$

$\varepsilon(\rho)/f \fallingdotseq 1.0$

In the above equation Y is the amplitude of the main subspan, that is correspondent to the amplitude at the center of the whole span.

3) Again, the equation to write the value of dynamic strains will be written as

$\varepsilon = f_0 \cdot Y_0$ when there is no damper.

and as

$\varepsilon = f \cdot [(\varepsilon_i(\rho)/f) \cdot Y]$ when there are dampers.

That is, when there is no damper the value of Y_0 is directly connected to the value of ε. But, when there are dampers the value of $(\varepsilon_i(\rho)/f) \cdot Y$ is directly connected to the value of ε. This is why the writer would like to call this product "Equivalent Amplitude" and to write αY, that is

$\alpha Y = (\varepsilon_i(\rho)/f) \cdot Y$

4) **a.** Concerning the balance amplitudes ($2Y_B$) the calculation with respect to the conductor IACSR120mm^2 for the domain of vibration frequency 0.5 ~ 150 Hz showed that the maximum value of energy balance amplitude ($2Y_B$) is equal to 13.7 mm for the vibration frequency of 28.75 Hz. And, this value of balance amplitude goes downward accompanied by the increase of the frequency.

b. As concerning the strain value (ε) there are two frequency bands in which the value of strain (ε) are rather high. And, these are 20~45 Hz and 75~90Hz.

c. The envelopes for both of above values ($2Y_B$ and ε) are plotted, in which there are also many scattered points bellow the envelopes.

5) Calculations were proceeded at the frequency intervals (Δf) of 5, 1.25, and 0.625 Hz.

For the frequency band 80~85 Hz, the higher value of the strains were,

178.1 MICRO	at 80.625 Hz	when	$\Delta f = 0.625$ Hz
226.3 〃	81.250 〃		$\Delta f = 0.625$ 〃
229.3 〃	82.550 〃		$\Delta f = 0.625$ 〃
197.3 〃	83.125 〃		$\Delta f = 0.625$ 〃
128.9 〃	85.043 〃		$\Delta f = 5.010$ 〃

And, the lower values of the strains were,

89.5 MICRO	at 80.033 Hz	when	$\Delta f = 5.010$ Hz
54.1 〃	81.875 〃		$\Delta f = 0.625$ 〃

In this report, more details with respect to the frequency intervals (Δf) were left for further study.

6) The value of dynamic strain (ε) is dependent on the value of strain coefficient ($\varepsilon_i(\rho)/f$). Consequently, the allowable maximum vibration amplitude on the conductor with dampers ($\overline{2Y_B}$) is sometimes greater than that on the conductor without damper ($\overline{\alpha Y}$), that is $\overline{2Y_B} > \overline{\alpha Y}$.

$\overline{2Y_B} > \overline{\alpha Y}$ is connected to $\varepsilon_i(\rho)/f < 1.0$

and on the other occasions, there is $\overline{2Y_B} < \overline{\alpha Y}$.

$\overline{2Y_B} < \overline{\alpha Y}$ is connected to $\varepsilon_i(\rho)/f > 1.0$

It is evident that the energy balance amplitude ($2Y_B$) for safety ought to be less than $\overline{2Y_B}$. When the dampers are installed, there is the chance that the increase of balance amplitude is likely to decrease the value of dynamic strain while amplitude keeps $\overline{2Y_B} > 2Y_B > \overline{\alpha Y}$.

And, the decrease of balance amplitudes are apt to increase the value of dynamic strain when $\overline{2Y_B} < 2Y_B < \overline{\alpha Y}$.

Thus it is very important that there is the following relation for safety ;

$2Y_B < \overline{2Y_B}$ when dampers are installed.

7) The preceding were all calculated with respect to ideal dampers. Some of the effects of the flexural rigidity of a conductor with dampers are clearly exploited.

Lastly, the writer thinks there are somethings not yet confirmed through the experiments especially for the higher domain of frequencies.

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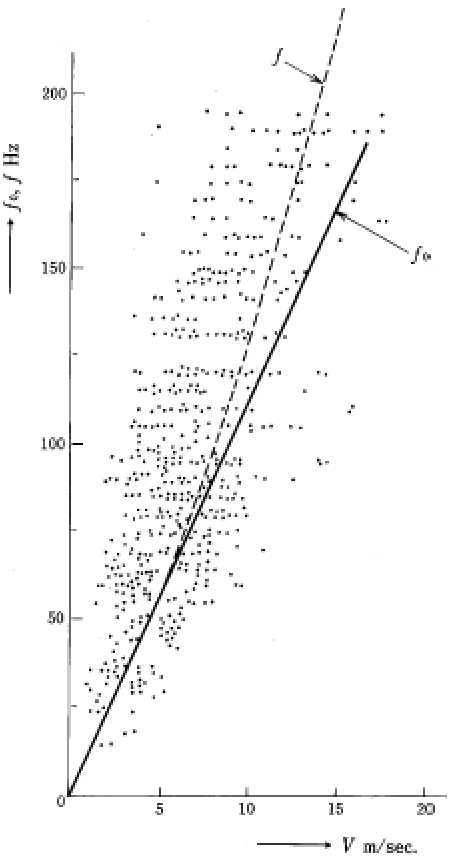


Fig. 1

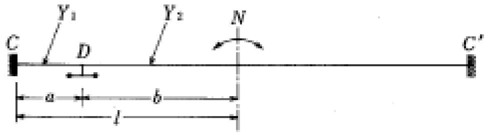
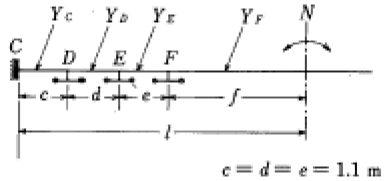


Fig. 2



$c = d = e = 1.1$ m
$f = 46.7$ m
$l = 50$ m

Fig. 3

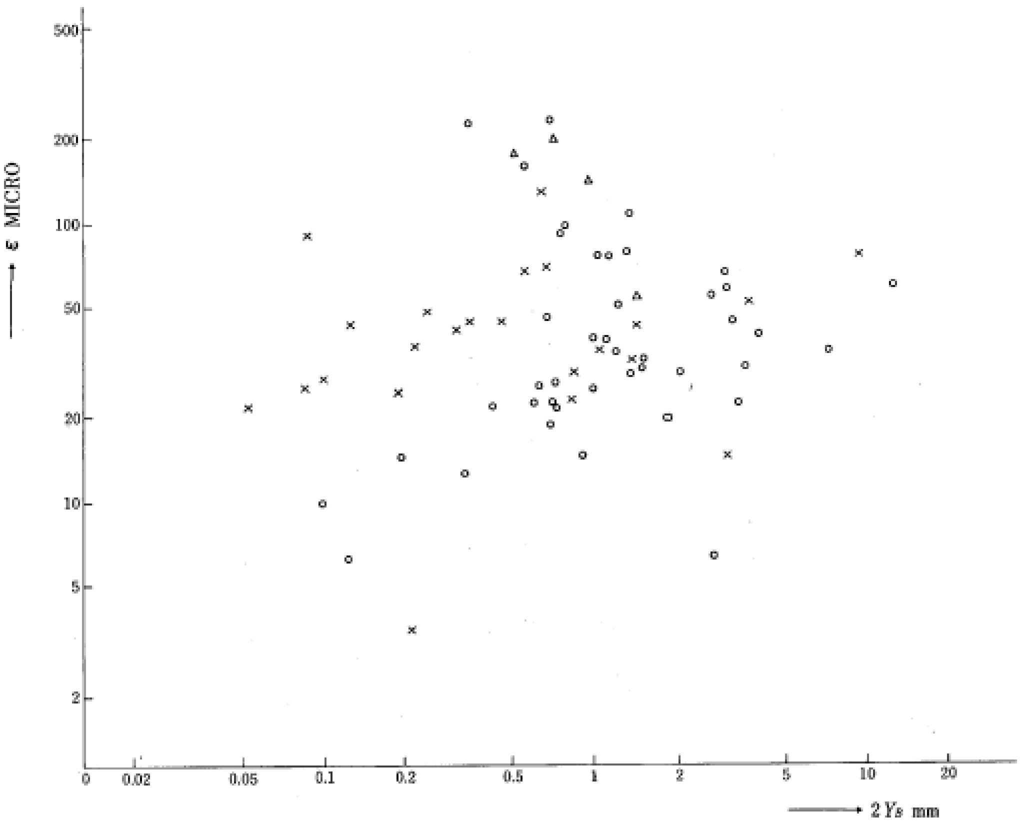


Fig. 4

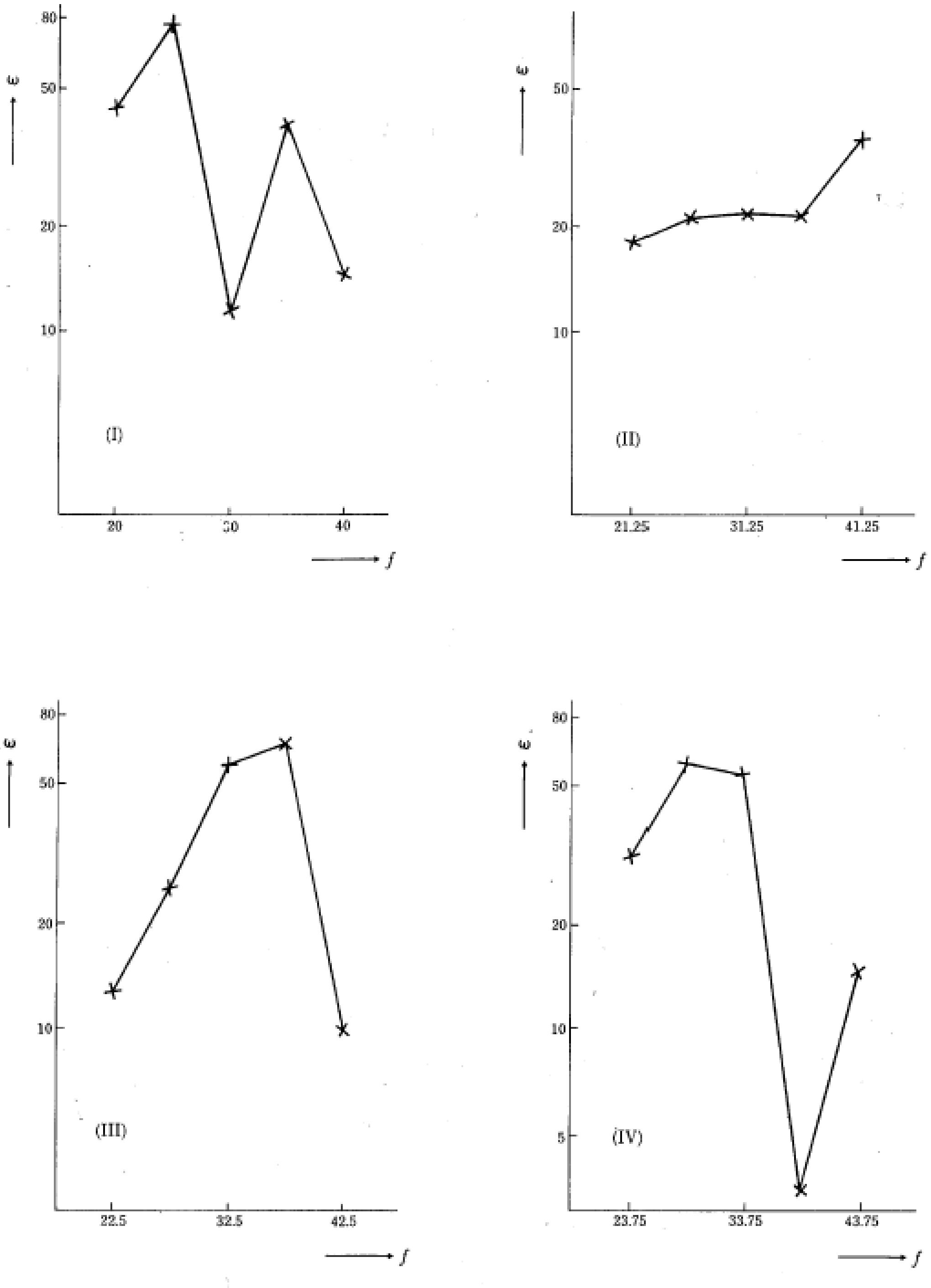
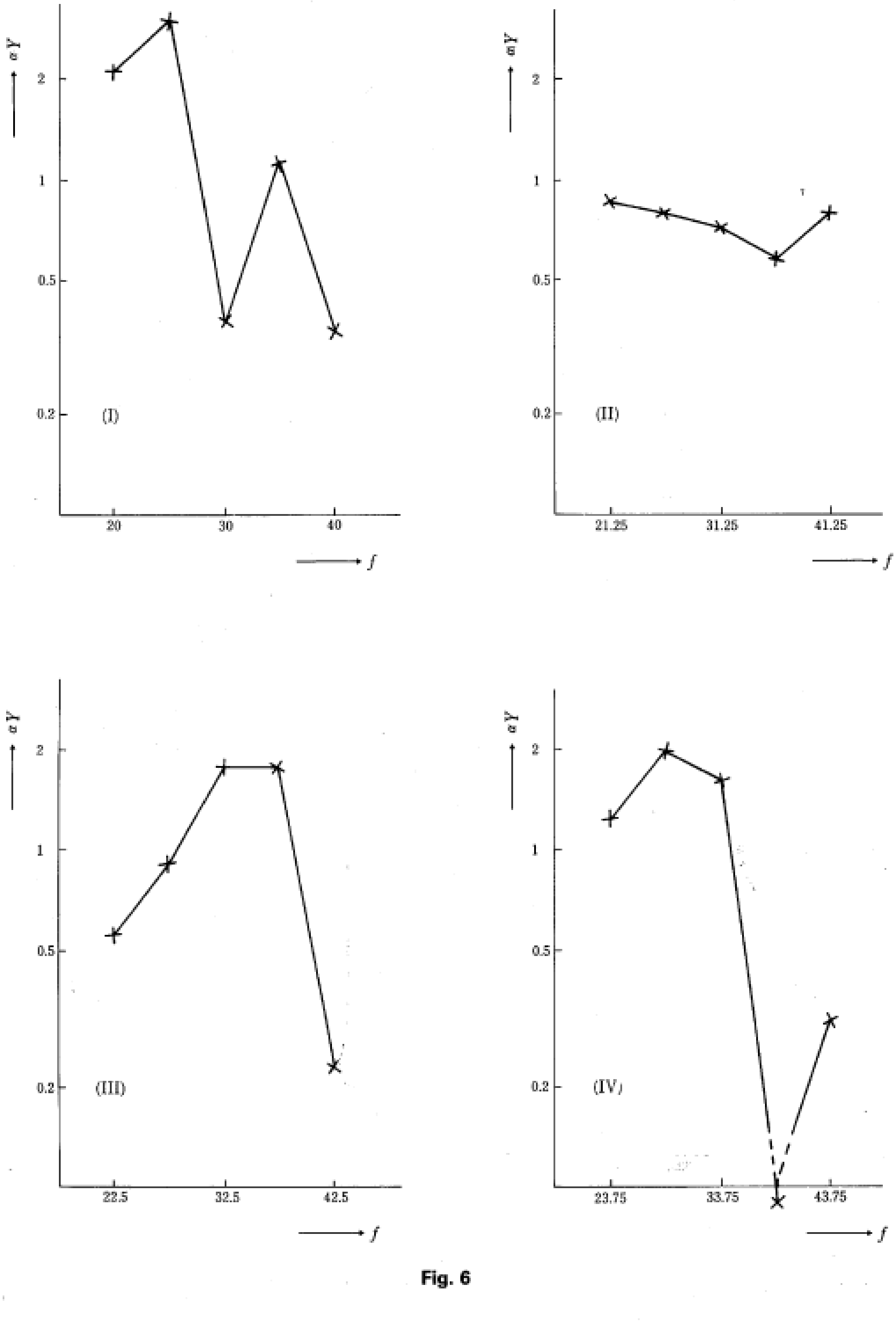
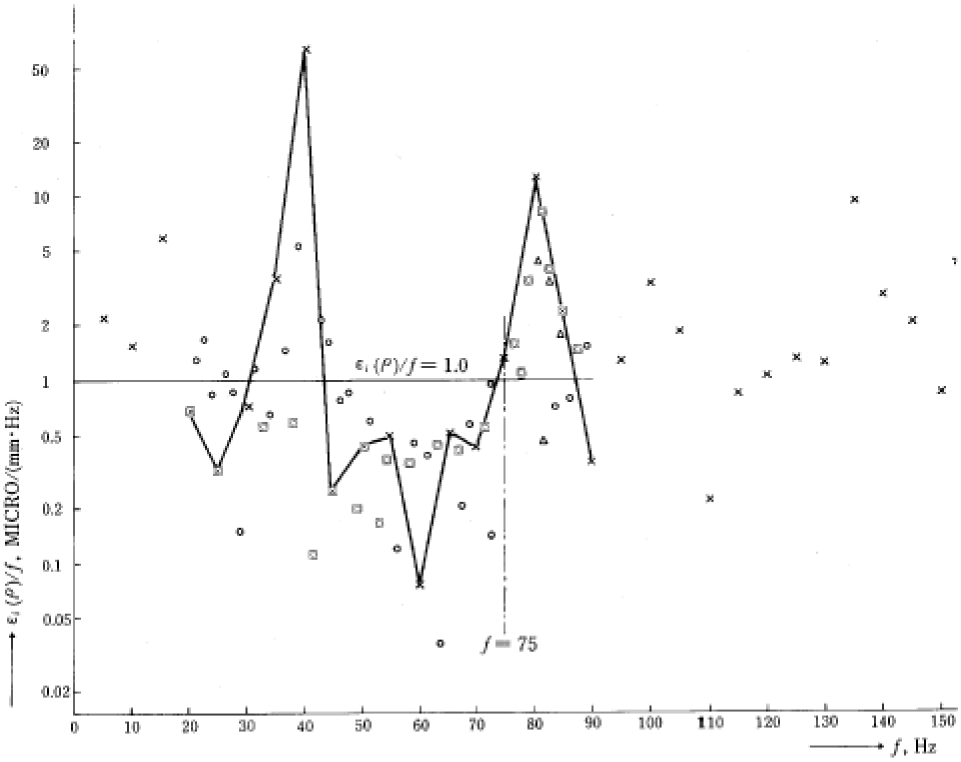

ε
80
50
20
10
(I)
20
40
f
ε
50
20
10
(II)
21.25
31.25
41.25
f
ε
80
50
20
10
(III)
22.5
32.5
42.5
f
ε
80
50
20
10
5
(IV)
23.75
33.75
43.75
f

Fig. 5



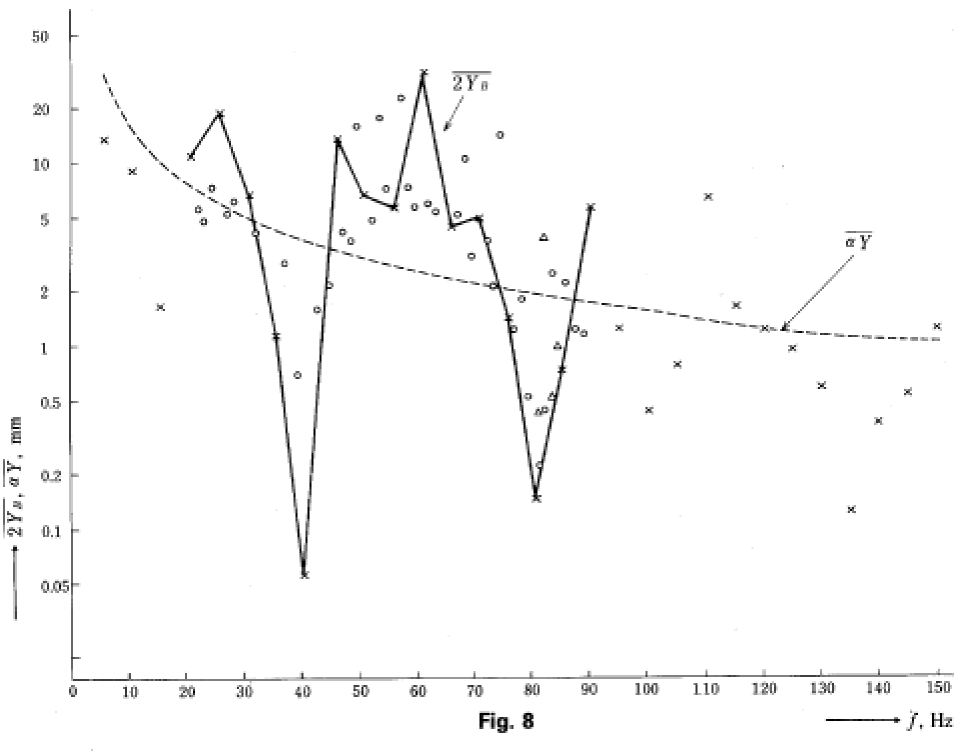
αY
2
1
0.5
0.2
(I)
20
30
40
f
(II)
21.25
31.25
41.25
(III)
22.5
32.5
42.5
(IV)
23.75
33.75
43.75

Fig. 6



50
20
10
5
2
1
0.5
0.2
0.1
0.05
0.02
$\varepsilon_i\,(P)/f$, MICRO/(mm·Hz)
$\varepsilon_i\,(P)/f = 1.0$
$f = 75$
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
f, Hz

Fig. 7



50
20
10
5
2
1
0.5
0.2
0.1
0.05
$\overline{2Y_B}$, $\overline{\alpha Y}$, mm
$\overline{2Y_B}$
$\overline{\alpha Y}$
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
f, Hz

Fig. 8

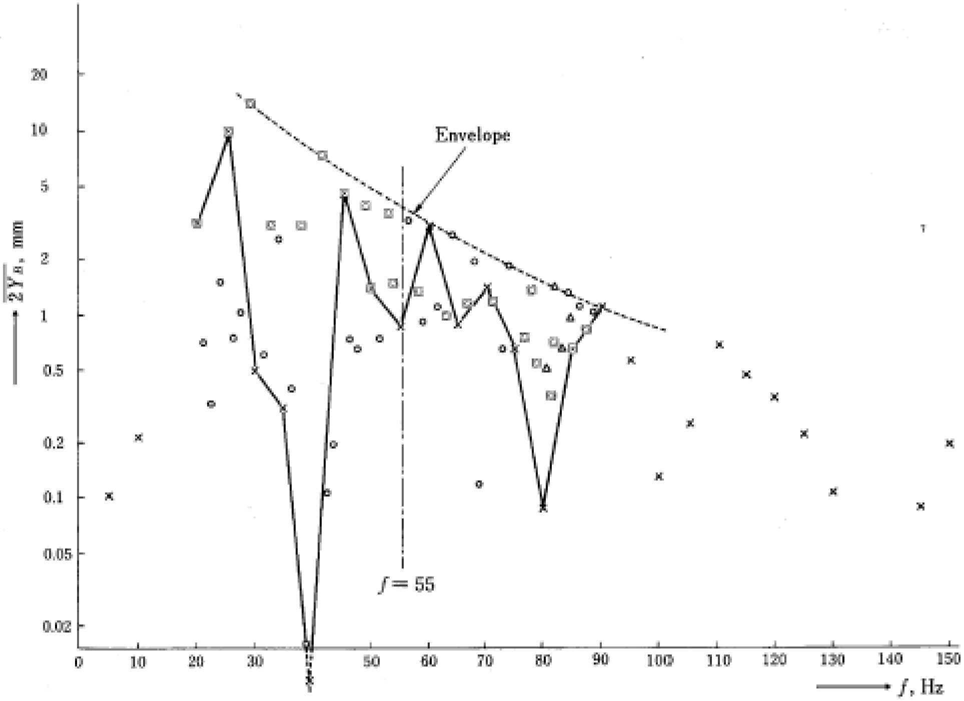

Envelope
f = 55
2Y_B, mm
f, Hz
20
10
5
2
1
0.5
0.2
0.1
0.05
0.02
0
10
20
30
50
60
70
80
90
100
110
120
130
140
150

Fig. 9

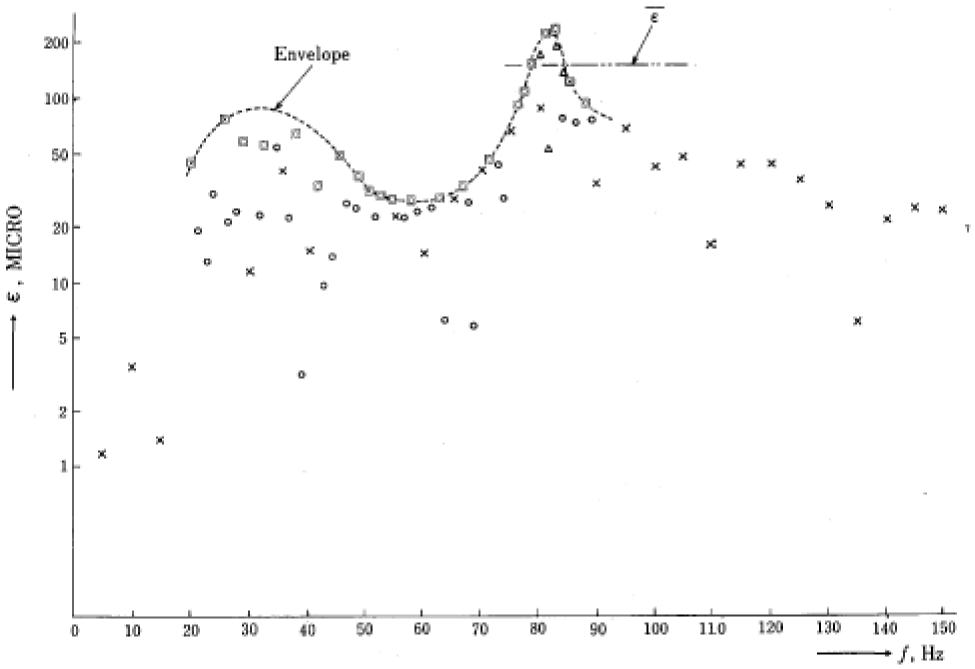


Fig. 10

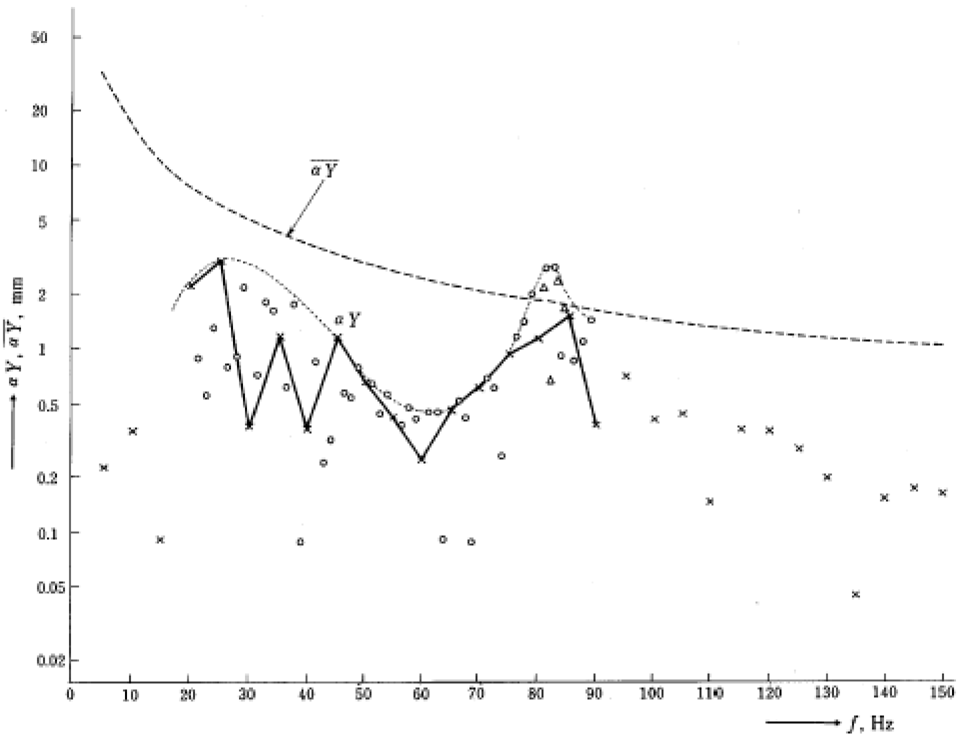
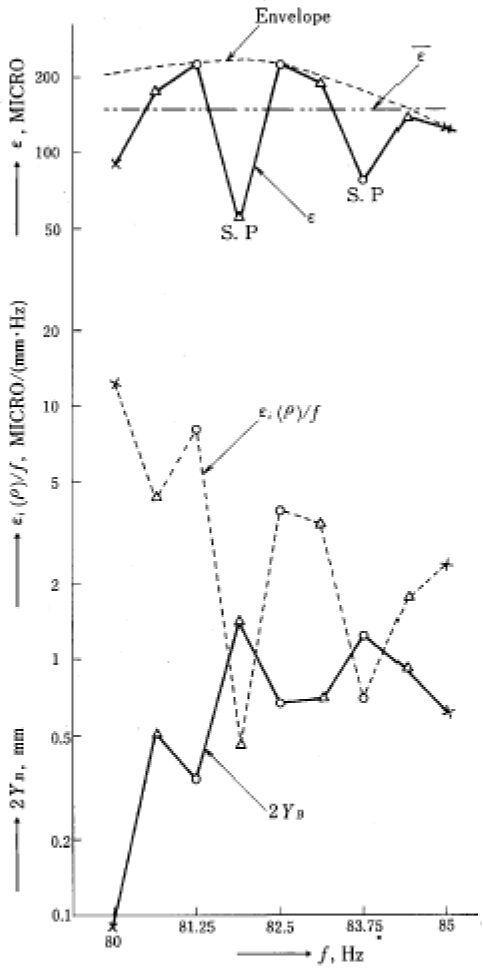


Fig. 11



Envelope
ε̄
200
100
50
20
10
5
2
1
0.5
0.2
0.1
→ ε, MICRO
→ $\varepsilon_i(\rho)/f$, MICRO/(mm·Hz)
→ $2Y_B$, mm
S. P
S. P
ε
$\varepsilon_i(\rho)/f$
$2Y_B$
80
81.25
82.5
83.75
85
→ f, Hz

Fig. 12

Appendix 1 : CALUCULATION FORMULAS

A) Numerical values

$D = 0.0175$ m
$M = 0.8456$ Kg/m
$S = 21854$ (20% UTS) N
$EI = 344.6$ N-m^2
$S/EI = 63.4185$ 1/m^2
$\sqrt{S/EI} = 7.9636$ 1/m
$\sqrt{EI/M} = 20.1872 = 2\pi \times 3.2129$ m^2/sec
$\sqrt{S \cdot M} = 135.94$ Kg/sec
$\sqrt{S/M} = 160.7618$ m/sec
$\mu_2 = 2\pi/\lambda$ 1/m
$\mu_1 = \sqrt{\mu_2^2 + S/EI} = \sqrt{\mu_2^2 + 63.4185}$ 1/m
$f = \mu_1 \cdot \mu_2 \cdot (\sqrt{EI/M}/2\pi) = 3.2129\mu_1 \cdot \mu_2$ Hz

B) Resonant Frequency

1. $A = \cos \mu_2 a - \cosh \mu_1 a \quad a = 1.1 \quad l = 50$
2. $B = \sin \mu_2 a - \frac{1}{\xi} \cdot \sinh \mu_1 a \quad \xi = \mu_1/\mu_2$
3. $F = \cos \mu_2(2a) - \cosh \mu_1(2a)$
4. $G = \sin \mu_2(2a) - \frac{1}{\xi} \cdot \sinh \mu_1(2a)$
5. $H = 2 \cosh \mu_1(2a)$
6. $I = \cos \mu_2(48.9)/\cos \mu_2(46.7)$
7. $J = -\cosh \mu_1(3a)$
8. $K = -\frac{1}{\xi} \cdot \sinh \mu_1(3a)$
9. $P = \cosh \mu_1 a + \sinh \mu_1 a$
10. $Q = \cos \mu_2(47.8)/\cos \mu_2(46.7)$
11. $K' = 7.9636\xi / [\mu_2 \cdot (1 + \xi^2)]$
12. $K'' = K'/\xi$
13. $T = F - K' \cdot A \cdot B$
14. $U = G - K' \cdot B^2$
15. $V = J - K'(AG + BF) + (K')^2 \cdot A \cdot B^2$
16. $W = K - K'(2BG) + (K')^2 \cdot B \cdot B^2$
17. $X = AI + TQ + V$
18. $Y = BI + UQ + W$
19. $Z(1) = X \cdot [2 \cdot \frac{1}{\xi} \cdot \sinh \mu_1(3a) - K''(BH + UP + W)]$
20. $Z(2) = Y \cdot [2 \cdot \cosh \mu_1(3a) - K''(AH + TP + V)]$

$Z(1) = Z(2)$ …… O.K. Go on !
$Z(1) \neq Z(2)$ …… N.G. Try again !

---

c) Wave coefficiant

21. $C_1 = 1.0$
22. $C_3 = -1.0$
23. $C_2 = -X/Y$
24. $C_4 = -C_2/\xi$
25. $D_1 = \cos \mu_2 a + C_2 \sin \mu_2 a$
26. $D_3 = -\cosh \mu_1 a + C_4 \sinh \mu_1 a$
27. $D_2 = -\sin \mu_2 a + C_2 \cos \mu_2 a - K'(D_1 + D_3)$
28. $D_4 = -\sinh \mu_1 a + C_4 \cosh \mu_1 a + \beta_1 \cdot K'' \cdot (D_1 + D_3)$
29. $E_1 = D_1 \cos \mu_2 b + D_2 \sin \mu_2 b \quad b = a$
30. $E_3 = D_3 \cosh \mu_1 b + D_4 \sinh \mu_1 b$
31. $E_2 = -D_1 \sin \mu_2 b + D_2 \cos \mu_2 b - K'(E_1 + E_3)$
32. $E_4 = D_3 \sinh \mu_1 b + D_4 \cosh \mu_1 b + \beta_2 \cdot K'' \cdot (E_1 + E_3)$
33. $F_1 = E_1 \cos \mu_2 e + E_2 \sin \mu_2 e \quad e = a$
34. $F_3 = E_3 \cosh \mu_1 e + E_4 \sinh \mu_1 e$
35. $F_2 = \tan \mu_2(46.7) \times F_1$
36. $F_4 = -F_3$

Appendix 2 : Table 4

f	ΔW—		
	D	*E*	*F*
20.04	1.147	0.624	0.392
21.25	0.227	0.091	0.120
22.50	0.110	0.030	0.078
23.75	0.707	0.121	0.623
25.00	4.803	0.445	4.886
26.25	0.384	0.015	0.424
27.50	0.548	0.004	0.789
28.75	3.311	0.002	3.737
30.05	0.124	0.002	0.611
31.25	0.513	0.028	0.486
32.50	3.395	0.389	2.709
33.75	3.304	0.642	2.059
35.03	0.348	0.103	0.150
36.25	0.545	0.222	0.150
37.50	4.949	2.655	0.623
38.75	0.013	0.009	0.001
40.08	0.233	0.196	0.000
41.25	1.337	1.329	0.052
42.50	0.111	0.129	0.018
43.75	0.222	0.296	0.087
45.01	2.862	4.279	2.033
46.25	0.749	1.266	0.849
47.50	0.704	1.311	0.585
48.75	1.520	3.090	3.418
50.02	1.012	2.222	2.981
51.25	0.480	1.126	1.749
52.50	0.835	2.081	3.686
53.75	0.797	2.089	4.140
55.00	0.457	1.250	2.665
56.25	0.389	1.103	2.609
57.50	0.604	1.764	4.465
58.75	0.422	1.260	3.379
60.02	0.132	0.402	0.978
61.25	0.472	1.440	4.202
62.50	0.438	1.341	4.020
63.75	0.020	0.061	0.185
65.00	0.401	1.190	3.705
66.25	0.503	1.463	4.529
67.50	0.306	0.861	2.641
68.75	0.013	0.035	0.104
70.00	0.542	1.370	4.024
71.25	0.629	1.485	3.918
72.50	0.555	1.130	2.913
73.75	0.071	0.137	0.331
75.00	0.751	1.241	2.718
76.25	1.052	1.428	2.576
77.50	1.146	1.186	1.616
78.75	1.874	1.298	0.996
80.03	0.452	0.156	0.010
81.25	2.031	0.151	0.639
82.50	1.318	0.059	3.333
83.75	0.086	0.075	0.810
85.04	2.365	0.608	3.779
86.25	0.010	0.390	1.809
87.50	0.000	1.004	3.353
88.75	0.022	2.658	0.786
90.07	0.007	0.251	0.597