# Approaches to "Turbulent-Flow-Vibration" ※

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## 1. Introduction

The purpose of this paper is to review the present knowledge with regard to the turbulent-flow-vibrations of overhead transmission line and to mention briefly the various things which might be of use for the further study of this kind of problems. Most of this paper is based on the literatures which seem of necessity to explain the turbulent-flow-vibrations and on the experiments carried out by the writers themselves.

Firstly, it is intended to mention about the fluctuation of parallel flow by the use of the concept of Markov chain from the statistical point of view to show that there is some periodic state. And also intended to mention that it is certain, as the results of another experiments worked out in the past, that Karman's vortex street under disturbance has the tendency to make an array of clouds and to decrease the frequency as much as one by integer times of the fundamental one.

Secondary, the writers' interests were paid to mention what is the source of pressure fluctuation on the bases of Navier-Stokes' theorem; and what is the energy-spectrum function correspondent to the prevailing wavenumber on the bases of Kolmogoroff spectrum law.

And lastly, some of the important things about the field experiments obtained by the writers will be presented.

We regret to say that this paper is too short to mention what is on our mind. And we hope that the literatures could be able to give more exact knowledge to those who wishes to study more about these problems.

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## 2. Fluctuation of Parallel Flow

### 2-1. Periodic State

We are specially interested in studying turbulent flow problems; where turbulance is of course to be considered as one of the random problems. And, it is of ncessity to fix the domain of space and time peculiar to the statistical circumstances under consideration.

The concept of "irreducible Markov chain" or "Markov chain in closed set" would be able to give us the exact knowledge. Markov chain gives the following definition.

"A sequence of trials with possible outcomes

$E_1$, $E_2$········will be called a Markov chain if the probabilities of sample sequences are defined by

$$P\{(E_{j_0}, E_{j_1}\cdots\cdots E_{j_n})\} = a_{j_0} p_{j_0 j_1} p_{j_1 j_2} \cdots\cdots p_{j_{n-2} j_{n-1}} p_{j_{n-1} j_n}$$

in terms of an initial probability distribution $\{a_k\}$ for the states $E_k$ at time 0 and fixed conditional probabilities $p_{jk}$ of $E_k$, given that $E_j$ has occured at the preceding trial."

A Markov chain is said to be irreducible if there exists no closed set other than the set of all states.

Moreover, this irreducible Markov chain has 4 states;

that is a. transient state··········································(1)

b. persistent state
- null state························(2)
- ergodic state ··················(3)
- periodic state···················(4)

(1), (2); not stationary. (3), (4); stationary in finite Markov chain.

Now, it is clear that we can find some periodic states even in the random phenomena of turbulence. (* 1)

### 2-2. Basic Equation

We begin by assuming that there is a laminar flow with some fluctuation which may be specified as

$$u = U(y) + \tilde{u}(x, y, t),$$

$$v = \tilde{v}(x, y, t),$$

$$p = P(x) + \tilde{p}(x, y, t)$$

Then, Navier-Stokes' theorem (＊2) will give, when some linearization is accepted as usual, the following result.

$$\tilde{p}_{xx} + \tilde{p}_{yy} = -2U_y \tilde{v}_x \quad \cdots\cdots (1)$$

where, $x, y$; cartesian coordinates

$t$; time

$p$; pressure

$u, v$; velocity component

$P$; mean pressure

$U$; mean velocity

$\tilde{u}, \tilde{v}, \tilde{p}$; fluctuation in respective quantities

$x, y$ in subscript; derivative

From Eq. (1) we can foresee that the product $U_y \tilde{v}_x$ is the source of pressure fluctuations. (＊3)

Because the values $\tilde{u}$, $\tilde{v}$ and $\tilde{p}$ are considered to be some periodic state as mentioned in the previous section and we are specially interested to study the effects of this kind of fluctuation to an aeolian vibration of a bare conductor which is likely to have the predominating effect of the prevailing frequency, it is advisable to proceed our way hereafter under the hypothesis that $\tilde{u}$, $\tilde{v}$ and $\tilde{p}$ are all sinusoidal in time and space coordinates.

## 3. Vortex under Disturbance

### 3-1. Abernathy and Kronauer's Study (＊3)

To study the nonlinear behavior of two adjacent layers of vorticity, Abernathy and Kronauer (1962) used a large computer to follow the motion of a number of point vortices. Typical results are shown in Fig. 1 where, at the first step, a little sinusoidal

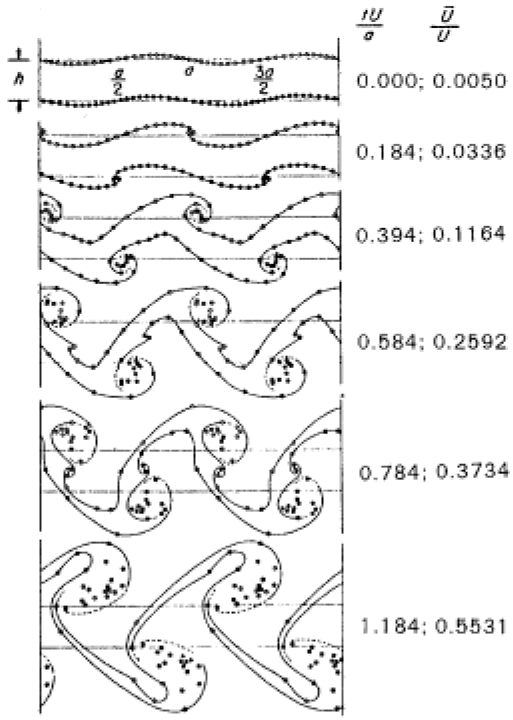


Fig. 1. Instability of a double layer of point vortices. (From Abernathy and Kronauer, 1962)

disturbance involved with the velocity inclination of parallel flow had been assumed.

It can be seen that the double layer evolves into a periodic array of clouds, which form a double alley of larger vortices. It must also be noted that the final double row is wider than the original double layer.

## 3-2. Nakagawa, Fujino, Arita and Shima's Study (＊4)

Wind tunnel test data for the power spectrum of the lift force acting on an oscillating cylinder are shown in Fig. 2. Where, we can find that the wind-excited force on an oscillating cylinder somewhat differes from that on the stationary one. This implies that there are some disturbances. The force could be considered mostly as that of some random excitation.

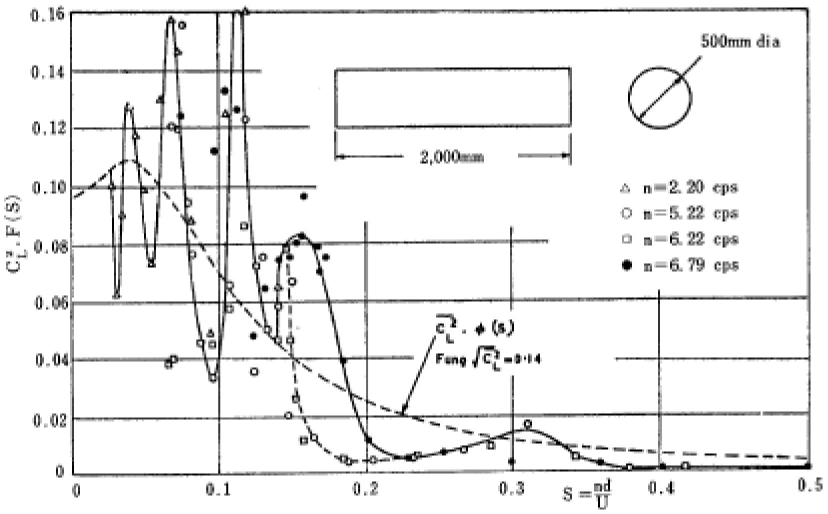


Fig. 2. Power spectrum of lift force for a bare cylinder
(From Nakagawa, Fujino, Arita and Shima, 1963)

The power spectrum gave predominant values when the Strauhal Number S is equal to

{0.3～0.4, 0.14～0.16, 0.11, 0.075,}

this values may be also considered to be nearly equal to

{$S_0$ $1/2 \cdot S_0$, $1/3 \cdot S_0$, $1/4 \cdot S_0$,}

if we put the original value $S=S_0$

About these results the authors told us that "the reason remains unsolved why such peaks can arise, and no successful theory has yet been found for this problem".

3-3. Some Comments on the Vortex Street

The writers' special interest is to find something which may come out of the above two studies.

Make attention to Fig. 3. If we reduce the number of vortices for an array of clouds in Fig. 1, then we shall have the equivalent Strauhal Number as

{$1/2 \cdot S_0$, $1/3 \cdot S_0$, $1/4 \cdot S_0$, etc.}

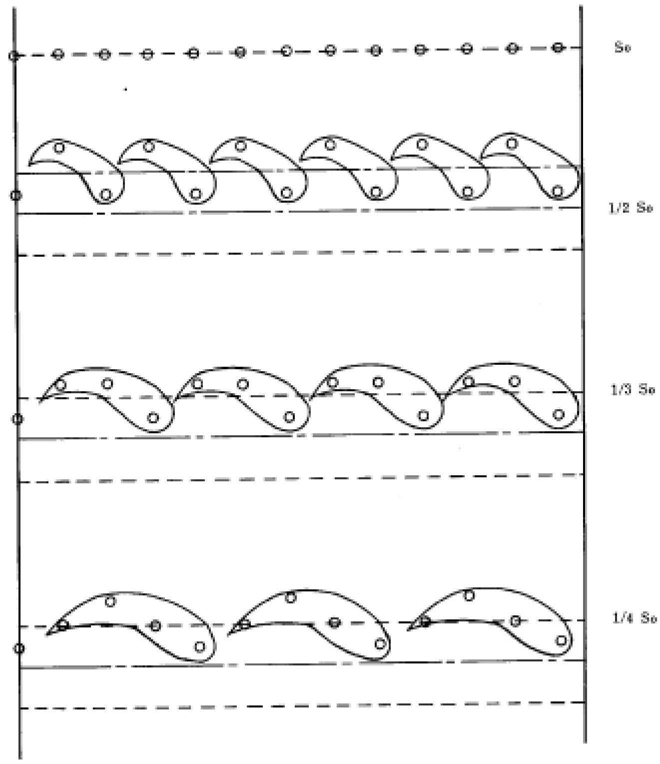


Fig. 3 Double Alloy of Vortices of Karman's Vortex Street under Turblence

With regard to these phenomena, we shall have the following criterion:

"When there are some disturbance, Karman's vortex street past a circular cylinder has the tendency to form a double alley of larger vortices and to decrease the vibration frequency of the circular cylinder as much as one by integer times of the fundamental one."

It is already mentioned that this disturbance is reasonably considered to be a sinusoidal one. And, when a slight disturbance of sinusoidal form gives rise to some degree of vertical vibration of a circular cylinder of which frequency is equal to that of the disturbance, the higher disturbance may come up resulting the higher vibration amplitude of the circular cylinder. Repetition of this phenomenon is most likely to give higher and higher vibration amplitude for the said prevailing frequency.

However, we cannot foresee exactly what is the frequency of this disturbance/ fluctuation; and we are going to draw our attention specially on this subjects in the following section.

## 4. Theory of Turbulence

### 4-1. Isotropic Turbulence

Turbulent fluid motion is an irregular condition of flow in which various quantities show a random variation with time and space coordinates. The scale of coordinates ought to be some suggested finite values correspondent to the turbulence under consideration. This concept is very important.

Of course there is a lot of type of turbulance, but the so-called "Isotropic Turbulence" is the simplest one, which means that the statistical features has no preference for any direction and a minimum number of quantities and relations are required to describe its structure and behavior.

Because of its relative simplicity it is more amenable to fundamental theoretical treatment than any other type of turbulence, and theoretical relations may be checked more easily by suitable experiments carried out in turbulence that is isotropic to a sufficient degree of approximation.

Hence many features of isotropic turbulence may apply to phenomenon in actual turbulence that is determined mainly by the fine-scale structure, where local isotropy prevails.

But even if we consider the non-isotropic large-scale structure of an actual turbulence, it is often possible to treat such a turbulence for purpose of a first approximation as if it were isotropic.

### 4-2. Kolmogoroff Spectrum Law

The description of the statistical behavior of an isotropic turbulence may be made by the application of the concept of the energy-spectrum function $E(k)$ as a function of wave number $k$. An energy-spectrum function $E(k)$ is demonstrated in Fig. 4.

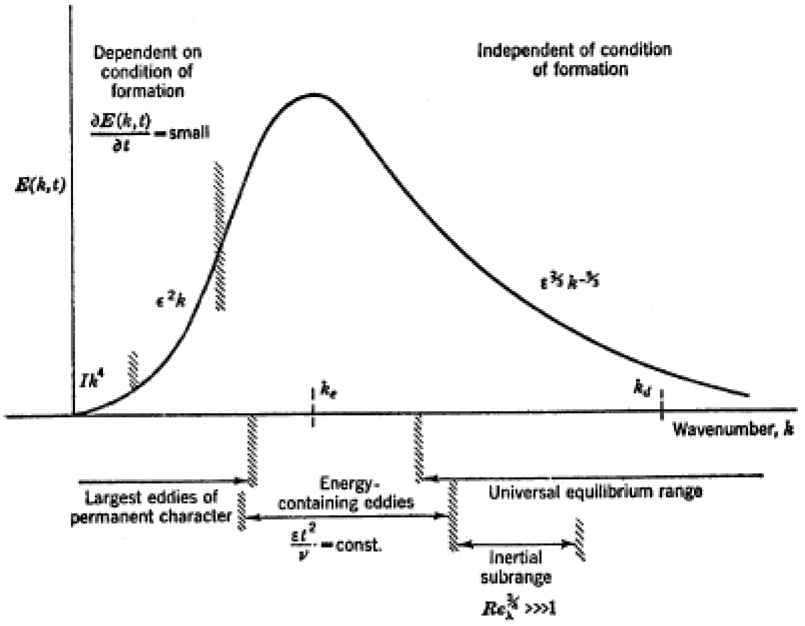


Fig. 4. Form of $E(k, t)$ in the various wavenumber ranges.

(From J. O. Hinze "Turbulence".)

We know that in the high-wavenumber range the turbulence may be considered statistically nearly steady and the rate of change of mean values may be regarded negligible. The character of the turbulence in this range is wholly determined by the energy flux through this range and the rate of dissipation.

These consideration have led Kolmogoroff to make the following hypothesis;

"At sufficient high Reynolds numbers there is a range of high wave numbers where the turbulence is statistically in equilibrium and uniquely determined by the parameters

$\varepsilon$ (dissipation by turbulence per unit mass)

and $\nu$ (kinematic viscosity)".

The dissipation takes place at all wavenumber but increases strongly as the wavenumber increases. In such a subrange the effect of the parameter $\nu$ would vanish, and the turbulence could be determined by the other parameter $\varepsilon$ alone.

Kolmogoroff has considered this subrange in his second hypothesis:

"If the Reynolds number is infinitely large, the energy spectrum in the subrange is independent of $\nu$, and is solely determined by one parameter $\varepsilon$".

Special attention may be drawn to the form of $E(k, t)$ in this subrange;

$$E(k, t) = \text{const.} \times \varepsilon^{2/3} k^{-5/3}$$

This form of $E(k, t)$, which applies at large Reynolds numbers, is frequently called the Kolmogoroff spectrum law, because Kolmogoroff was the first arrived to this result. (＊5)

Experiments carried out by A. G. Davenport (＊6) may be said to have a good agreement with the Koimogoroff spectrum law as was suggested by O. Sakai (＊7).

Hereby, we could catch the information about the frequency of turbulence in the term of wavenumber of which energy-spectrum may be regarded to agree with that of Kolmogoroff's spectrum. And, after a little calculation we had th following equation(＊7)

$$f = \text{const.} \times (V/D)^{1/4}$$

This equation is true only when the Kolmogoroff spectrum law is to be applied without trouble.

## 5. Field Experiments

### 5-1. Terrain Conditions

Aeolian vibration under turbulent flow such as mentioned in the previous sections had been experienced mainly around the mountanious terrain.

This is why the product $U_y, \bar{v}_x$ is certainly liable to grow up as high as sufficient to produce the pressure fluctuation. That is, the convergent stream of the air around the top of the mountain gives higher $U_y$, and the turbulence around there gives higher $\bar{v}_x$. This $\bar{v}_x$ is expected to be a sinusoidal function with respect to time an space coordinates.

Of course this kind of turbulence is nothing but a hypothetical consideration because no turbulence is a true sinusoidal function, but the frequency locking effect of the circular cylinder for the prevailing frequency of the turbulence will lead us up to the conclusion that the above consideration is one of the good idea to catch th first approach of the existing phenomenon.

Besides, experiments told us that the turbulent-flow-vibrtions are liable to be seen when it rains, however the reasons are rather ambiguous.

Remember that the dissipation by turbulence plays an important role involved with the energy spectrum of the turbulence. Does the rain increase the dissipation ?

## 5-2. Accelerometer Patterns

A typical pattern of the conductor vibration under turbulence picked up by an accelerometer at Atami Line in Japan is shown in Fig. 5a. The higher frequency is

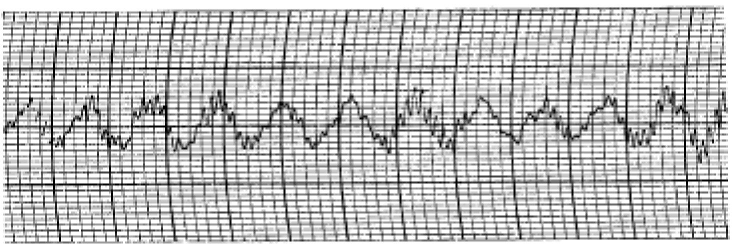

Fig. 5a Accelerometer pattern of turbulent-flow-vibration at Atami Line (ACSR; 120mm$^2$, D=16.1mm)

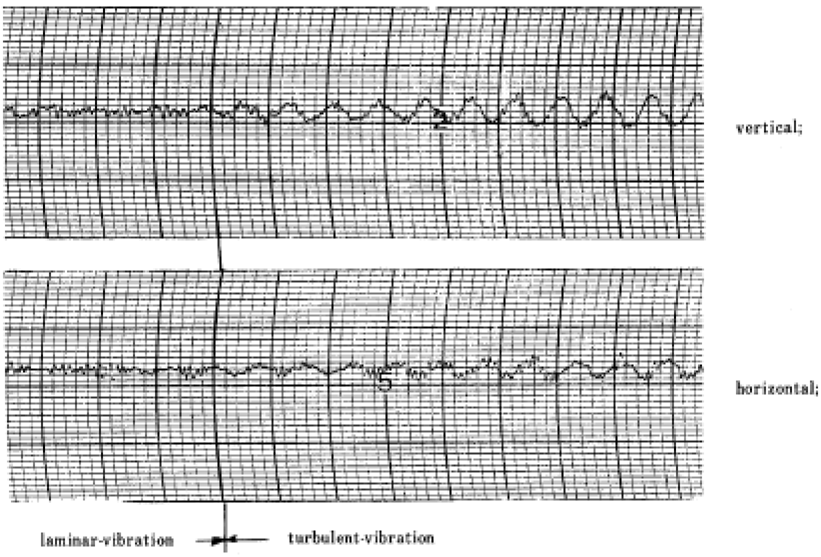


Fig. 5b Accelerometer pattern of turbulent-flow-vibration at Kumano Line (GSW; 90mm$^2$, D=12mm)

correspondent to the vibration of Karman's vortex street for laminar flow, and the lower frequency to that of turbulent flow.

Another typical component patterns observed at Kumano Line in Japan are shown in Fig. 5b; where, component patterns mean both of the vertical component and the horizontal component of a single turbulent-flow-vibration.

## 6. Conclusion

1) From the theoretical point of view, we have no hesitation to say that we must have some finite valuses of $U_y$, $\bar{p}_x$ and $\varepsilon$ for the "Tarbulent-Flow-Vibration".
2) Experiments told us that this kind of vibration is liable to be seen at the mountainious region, where $U_y$, $\bar{v}_x$ and $\varepsilon$ seem to be predominant.
3) Of course these phenomena are non-linear, but it is possible to make some hypotheses and manage them as though they are linear phenomena to catch the first approach of them.
4) Criterion mentioned in section 3-3 may be said to be safe, refered to the study of the sections 3-1 and 3-2. And, this may be truly experienced when Karman's vortex street is exposed to a quasi-sinusoidal disturbance.
5) A typical accelerometer pattern of the conductor vibration under turbulance has a little trace of laminar-flow-vibration.
6) These trends may be considered to be true, but we cannot predetermine the terrain and other conditions before experience. These problems are left for the future work as well as the problems of distribution of turbulences along the span.

## 7. Acknowledgement

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## 8. Literatures


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